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Letter to the Editor

On symmetry of vibrational floating

S.A. Gerasimov

Department of General Physics, Faculty of Physics, Rostov State University, Zorge ave, 5, Rostov-on-Don 344090, Russia

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1. Introduction

Motion of a body with non-zero average velocity caused by actions undirected on the average is called vibrational transposition. Such a definition formulated by Blekhman [1] corresponds to the theory based on a model of a body located on a vibrating rough surface [1]. This model is said to be also valid for the case of motion caused by oscillations of internal mass in presence of asymmetry of frictional or resistance forces [2,3]. An example of this type of motion is the vibrational transposition in a liquid [4]. Unfortunately, this kind of motion is not realistic enough. At low velocities of a system in a fluid, the resistance force is proportional to the speed but not to the square of the velocity [5]. Therefore, vibrational floating with the magnitude of the resistance force increasing linearly with speed has remained without satisfactory solution. So far one does not know how fast the vibrational propulsive device can move in a fluid. One should remember that the average velocity of the vibrational transposition depends on the type of oscillations. By the reasons mentioned above, one cannot immediately use known results [1,5,6]. Therefore, in order to avoid ambiguity one should echo the framework of this vibrational transposition in detail. Especially, as it is found out that this is neither more nor less than the law of motion of the center of mass of the system. The problem is to establish the connection between the parameters describing the system but not to develop the theory of the vibrational transposition. Below is a simple interpretation of the theory of vibrational floating [1,4].

2. Vibrational transposition

Consider an asymmetrical platform P of mass M inside which the block B of mass m executes forced oscillations in the horizontal direction. The platform is partly submerged in a liquid L, as shown in Fig. 1, with a resistance force $\mathbf{F}_r$ proportional to the velocity of motion $\mathbf{v}$. The location of the block B in the inertial frame of reference can be determined by the sum of two vectors $\mathbf{x}+\mathbf{x}_m$ the first of which $\mathbf{x}$ describes the position of the platform with respect to the inertial frame of

E-mail address: gsimp@yandex.ru (S.A. Gerasimov).

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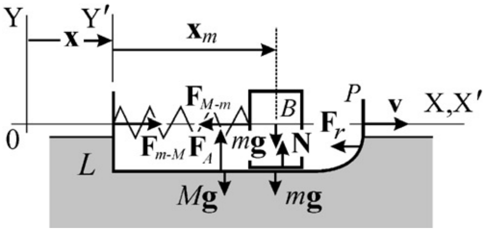

Fig. 1. Illustration of the vibrational floating.

references XOY and the second vector $\mathbf{x}_m$ describes the position of the load B with respect to the platform P. The forces acting on the mass m are the normal $\mathbf{N}$ and the horizontal forces $\mathbf{F}_{M-m}$ due to the box and the force of gravity $m\mathbf{g}$. The forces acting on the platform P are the vertical $m\mathbf{g}$ and horizontal forces $\mathbf{F}_{m-M}$ exerted on the platform by the load of the mass m, the force of gravity $M\mathbf{g}$, the Archimedes' force $\mathbf{F}_A$ and the resistance force $\mathbf{F}_r$. Since the mass m and the platform P do not move in vertical direction then the sum of the forces $\mathbf{N}$ and $m\mathbf{g}$ and the sum of the forces $\mathbf{F}_A$, $M\mathbf{g}$ and $m\mathbf{g}$ equal zero. Thus, equations of motion of the platform and the load are

$$M\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} = \mathbf{F}_r + \mathbf{F}_{m-M}, \tag{1}$$

$$m\frac{\mathrm{d}^2(\mathbf{x}+\mathbf{x}_m)}{\mathrm{d}t^2} = \mathbf{F}_{M-m}. \tag{2}$$

The forces $\mathbf{F}_{M-m}$ and $\mathbf{F}_{m-M}$ are the internal ones. Their sum equals zero. By combining Eqs. (1) and (2), the equation of motion can be written as

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}[M\mathbf{x} + m(\mathbf{x}+\mathbf{x}_m)] = \mathbf{F}_r, \tag{3}$$

which describes the vibrational transposition of the platform in the liquid for any given character $x_m(t)$ of oscillations of the load B. The term in the square brackets is nothing else but a vector describing the center of mass of the system of the bodies P and B.

The behavior of the drag force F_r depends on many things, including the shape of the object, the velocity v=dx/dt of the object relative to the fluid, the direction of motion, and the nature of the fluid:

$$\mathbf{F}_r = \begin{cases} -\lambda_> v, & v > 0, \\ -\lambda_< v, & v < 0, \end{cases} \tag{4}$$

where $\lambda_>$ and $\lambda_<$ are proportionality constants corresponding to the motion in positive ($v>0$) and negative ($v<0$) directions of the X-axis. Expression (4) is approximately valid when the magnitude of v is not large. For the case of the harmonic oscillations of the load m

$$x_m = a\cos\frac{2\pi t}{T}. \tag{5}$$

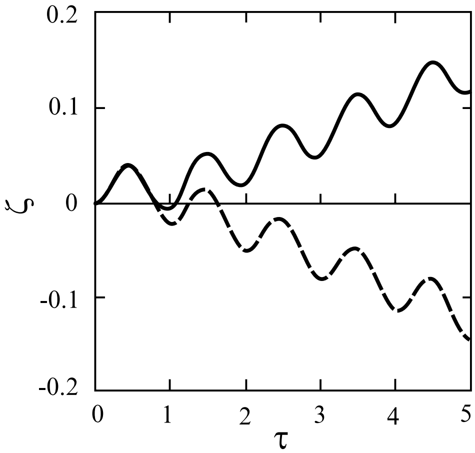

Fig. 2. The path $x = 4\pi^2 ma\zeta/M_0$ travelled by the vibratory propulsive device during time $t = M_0\tau/\lambda_>$ at the period of vibrations T equal $M_0/\lambda_>$. Solid line is for $\delta = 2$ and dashed line is for $\delta = \frac{1}{2}$.

When $a > 0$, the equation of motion (3) can be rewritten in the form

$$\frac{d^2\zeta}{d\tau^2} + \left(\frac{(1-\delta)}{2}\,\mathrm{sign}\left(\frac{d\zeta}{d\tau}\right) + \frac{(1+\delta)}{2}\right)\frac{d\zeta}{d\tau} - \frac{1}{\theta^2}\cos\frac{2\pi\tau}{\theta} = 0, \tag{6}$$

where $\mathrm{sign}(\vartheta) = 1$ if $\vartheta > 0$ and $\mathrm{sign}(\vartheta) = -1$ if $\vartheta < 0$. It turns out that introducing dimensionless variables

$$\zeta = \frac{M_0}{4\pi^2 ma}x, \quad \tau = \frac{\lambda_>}{M_0}t, \quad \vartheta = \frac{M_0^2}{4\pi^2 ma\lambda_>}\frac{dx}{dt} \tag{7}$$

and

$$\theta = \frac{\lambda_>}{M_0}T, \quad \delta = \frac{\lambda_<}{\lambda_>}, \tag{8}$$

one may decrease the number of variables. Here, $M_0 = M + m$ is the total mass of the system of the bodies and T is the period of the vibrations of the load B. Now the reduced path ζ travelled by the system depends only on the reduced time τ, the reduced period θ and the parameter of asymmetry δ. One can solve Eq. (6) numerically for any given parameters θ and δ. An example of such calculations is shown in Fig. 2. But the problem of the scaling has not been solved. Too many variables describing the process of such a motion.

3. Symmetry

The average reduced velocity $\langle\vartheta\rangle = \langle d\zeta/d\tau\rangle$ depends only on θ and δ. When the reduced period θ is small ($\theta < 1$) and $\delta \gg 1$, the value $\langle\vartheta\rangle/\delta$ depends only on $\delta\theta$. The case of the irreversible transposition of the platform in the direction opposite to the X-axis corresponds to the condition $\delta \ll 1$. This means that for small δ when $\theta < 1$, the average reduced velocity $\langle\vartheta\rangle$ depends only on the reduced period θ. On the other side, Eq. (6) contains the parameter of asymmetry δ only in the

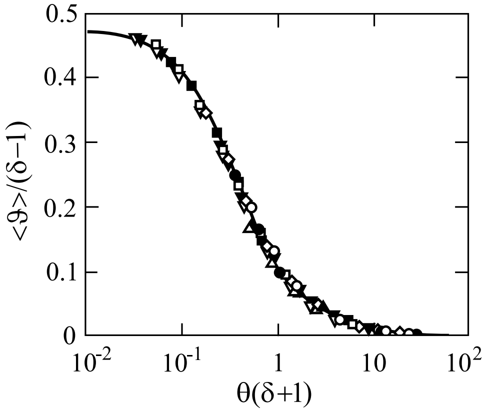

Fig. 3. The scaled dependence of the average reduced velocity of the vibrational floating $\langle \vartheta \rangle$ versus the reduced period of vibrations θ and the parameter of asymmetry δ at $\theta < 1$. Points ($\Delta - \delta = 1.1$, $\nabla - \delta = 2$, $\square - \delta = 4$, $\diamond - \delta = 8$, $\bigcirc - \delta = 16$) are the results of solving the equation of motion (6) for the harmonic oscillations (open points) and the piece-wise constant force (solid points). The curve is relation (9).

forms $(\delta - 1)$ or $(\delta + 1)$. This symmetry enables one to write universal variables of the average velocity and the period of vibrations. Indeed, the simultaneous transformations $\lambda_> \to \lambda_<$ and $\lambda_< \to \lambda_>$ correspond to a turn of the platform in the opposite direction $\langle v \rangle \to -\langle v \rangle$ with the same magnitude of the average velocity. The value $\langle \vartheta \rangle/(\delta - 1)$ is anti-symmetrical with respect to the simultaneous transpositions $\lambda_> \leftrightarrow \lambda_<$ but the value $\theta(\delta + 1)$ saves the value of the period T at these transformations. Thus, a universal dependence, if it exists, could be $\langle \vartheta \rangle/(\delta - 1) = f(\theta(\delta + 1))$ with some function f. Fig. 3 shows the results of numerical calculations plotted using the mentioned variables.

For practical purposes, it is useful to have an approximate dependence of $\langle \vartheta \rangle/(\delta - 1)$ on $\theta(\delta + 1)$. Satisfactory results are obtained by means of fitting a formula $\langle \vartheta \rangle/(\delta - 1) = (A + B\theta^{3/2}(\delta + 1)^{3/2})^{-2/3}$ to the calculation results presented in Fig. 3. The usual least-square method gives $A \approx (40/19)^{3/2}$ and $B \approx 8A$. In the usual notations, the average velocity as a function of all parameters of the system is

$$\langle v \rangle \approx \frac{19\pi^2 ma(\lambda_< - \lambda_>)}{10M_0[M_0^{3/2} + 8T^{3/2}(\lambda_< + \lambda_>)^{3/2}]^{2/3}}. \tag{9}$$

It is seen that this procedure of scaling does not depend on the type of oscillations. It would be a good thing, therefore, to apply this method to the case of oscillations produced by the piece-wise constant force:

$$m\frac{d^2 x_m}{dt^2} = -\frac{4\pi^2 m}{T^2} a_c \operatorname{sign}\left(\cos\frac{2\pi t}{T}\right). \tag{10}$$

In this case, it turns out that the scaled dependence of the average velocity upon the period is almost identical to the described one if $a_c \approx 0.81a$. This is also shown in Fig. 3.

4. Conclusions

Note that the maximum velocity of vibrational floating corresponds to the case of irreversible motion when $\delta \gg 1$. On the other hand, it follows from Eq. (9) that the maximum value of $\langle \vartheta \rangle/(\delta - 1)$ is about 0.5. This means that as the reduced period of the vibrations approaches zero, the average velocity approaches the asymptotic value

$$\langle v \rangle_{max} \approx 2\frac{\pi^2 ma\lambda_<}{M_0^2} \tag{11}$$

and does not depend on $\lambda_>$ since $\lambda_< \gg \lambda_>$. When $\lambda_< = 1$ kg/s, $M_0 = 1$ kg, $a = 0.1$ m, $m/M = \frac{1}{2}$, this value is $\langle v \rangle_{max} \approx 1$ m/s. At large θ and δ, the average reduced velocity $\langle \vartheta \rangle$ is inversely proportional to the square of the reduced periods and does not depend on the parameter of the asymmetry: $\langle \vartheta \rangle \approx 0.32/\theta^2$ [7]. To take this fact into account, one may add the corresponding item in the denominator of Eq. (9). It gives

$$\langle \vartheta \rangle \approx \frac{19(\delta - 1)}{40[1 + 8\theta^{3/2}(\delta + 1)^{3/2} + 9\delta^{3/2}\theta^3/5]^{2/3}}. \tag{12}$$

One should remember that transformations of symmetry can be exact only in asymptotic fields. The problem has been solved. Apart from estimating the average velocity of vibrational floating, the distinction and likeness of two types of oscillations in this kind of motion are mentioned.

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